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$F^0 - \bar{F}^0$ Mixing and CP Violation in the General Two Higgs Doublet Model

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Abstract

A phenomenological analysis of the general two Higgs doublet model is presented. Possible constraints of the Yukawa couplings are obtained from the $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ mixings. A much larger $D^0 - \bar{D}^0$ mixing than the standard model prediction is possible. It is shown that the emerging of various new sources of CP violation in the model could strongly affect the determination of the unitarity triangle. It can be helpful to look for a signal of new physics by comparing the extracted angle β from two different ways, such as from the process $B \to J/\psi K_S$ and from fitting the quantities $|V_{ub}|$, Δm_B and ϵ.

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I. INTRODUCTION

In the standard model(SM) of an electroweak $SU(2)_L \times U(1)_Y$ gauge theory with only one Higgs doublet, the only source of CP violation comes from the complex Yukawa coupling between Higgs and fermion fields [1]. Since the Higgs sector of SM is not well understood yet, many possible extensions of SM have been proposed [2]. One of the simplest extensions of the SM is to simply add one Higgs doublet without imposing the *ad hoc* discrete symmetries. For the convenience of mention in our following discussions, we may call such a minimal extension of the standard model with generally adding an extra Higgs doublet as an S2HDM [3–11] and assume CP violation solely originating from the Higgs potential [12,7,8]. The most general Yukawa coupling and Higgs potential can be written as:

$$L_Y = \bar{Q}_L(\Gamma_1^U \tilde{\phi}_1 + \Gamma_2^U \tilde{\phi}_2)U_R + \bar{Q}_L(\Gamma_1^D \phi_1 + \Gamma_2^D \phi_2)D_R \tag{1}$$

and:

$$\begin{aligned} V(\phi_1, \phi_2) = & -\mu_1^2 \phi_1^\dagger \phi_1 - \mu_2^2 \phi_2^\dagger \phi_2 - (\mu_{12}^2 \phi_1^\dagger \phi_2 + h.c) \\ & + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2) + \lambda_4 (\phi_1^\dagger \phi_2 \phi_2^\dagger \phi_1) \\ & + \frac{1}{2}[\lambda_5 (\phi_1^\dagger \phi_2)^2 + h.c] + [(\lambda_6 \phi_1^\dagger \phi_1 + \lambda_7 \phi_2^\dagger \phi_2)(\phi_1^\dagger \phi_2) + h.c] \end{aligned} \tag{2}$$

The major issue with respect to the two Higgs doublet model is that it allows flavor changing neutral current (FCNC) at tree level, which must be strongly suppressed in $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing processes. In order to prevent FCNC from tree level, an *ad hoc* discrete symmetry is often imposed

$$\begin{aligned} \phi_1 \to -\phi_1 \text{ and } \ \phi_2 \to \phi_2 \\ U_{R_i} \to -U_{R_i} \text{ and } \ D_{R_i} \to \mp D_{R_i} \end{aligned} \tag{3}$$

Thus, one obtains the so called mode I and model II, which depends on whether the up-type and down-type quarks are coupled to the same or different Higgs doublet respectively [2]. Once the discrete symmetry is adopted, the factor μ_{12}, λ_6 and λ_7 in Eq.(2) mush vanish. As a result, no CP violation can occur from $V(\phi)$. Thus the only source of CP violation is the complex Yukawa couplings, which lead to a phase in the Cabibbo-Kobayashi- Maskawa(CKM) quark mixing matrix.

In contrast, one can replace the discrete symmetry by an approximate global family symmetry [4,5,7,8] , thus the suppression of FCNC can be explained via the smallness of the off-diagonal terms. Furthermore, when abondoning the discrete symmetries, one can obtain, after spontaneous symmetry breaking, rich sources of CP violation from a single relative phase between the two vacuum expectation values of Higgs fields. It has been shown [7,8] that even when the CKM matrix is real, the single phase arising from the spontaneous symmetry breaking can provide enough CP violation to meet the experimental measurments. One of particular important observations is a new source of CP violation in charged Higgs boson interactions, which is independent of the CKM phase and can lead to a value of ϵ'/ϵ as large as 10^{-3} [5]. In the S2HDM, the two Higgs fields have, in general, the vacuum expectation values:

$$
\begin{aligned}
< \phi_1^0 > &= \frac{v}{\sqrt{2}} \cos\beta e^{i\delta} \\
< \phi_2^0 > &= \frac{v}{\sqrt{2}} \sin\beta
\end{aligned} \tag{4}
$$

It is natural to use a suitable basis:

$$
\begin{aligned}
H_1 &= \cos\beta\phi_1 e^{-i\delta} + \sin\beta\phi_2 \\
H_2 &= \sin\beta\phi_1 e^{-i\delta} - \cos\beta\phi_2
\end{aligned} \tag{5}
$$

such that:

$$
\begin{aligned}
H_1 &= \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+\rho) \end{pmatrix} \\
H_2 &= \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(R+iI) \end{pmatrix},
\end{aligned} \tag{6}
$$

where H^0, R, I are real Higgs bosons. The three neutral scalars $\hat{H}_k^0 \equiv (R, \rho, I)$ can be rotated into mass engenstates $h_k^0 \equiv (h, H^0, A)$ via orthogonal matrix O^H.

$$
\hat{H}_k^0 = O_{kl}^H H_l^0 \tag{7}
$$

From approximate global family symmetries, we know that the Yukawa coupling matrices Γ_i^F in Eq.(1) have small off-diagonal elements, typically between 0.01 and 0.2 in order to meet the constraint of FCNC from $K^0 - \bar{K}^0, B^0 - \bar{B}^0$ mixing. The Yukawa interaction can be rewritten as [7]:

$$
L_Y = (L_1 + L_2)(\sqrt{2}G_F)^{1/2} \tag{8}
$$

with

$$
\begin{aligned}
L_1 = \sqrt{2}\Bigg(& H^+ \sum_{i,j}^{3} \xi_{d_j} m_{d_j} V_{ij} \bar{u}_L^i d_R^j - H^- \sum_{i,j}^{3} \xi_{u_j} m_{u_j} V_{ij}^\dagger \bar{d}_L^i u_R^j \Bigg) \\
& + H^0 \sum_{i}^{3} (m_{u_i} \bar{u}_L^i u_R^i + m_{d_i} \bar{d}_L^i d_R^i) \\
& + (R+iI) \sum_{i}^{3} \xi_{d_i} m_{d_i} \bar{d}_L^i d_R^i + (R-iI) \sum_{i}^{3} \xi_{u_i} m_{u_i} \bar{u}_L^i u_R^i + h.c
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
L_2 = \sqrt{2}\Bigg(& H^+ \sum_{i,j' \neq j}^{3} V_{ij'} \mu_{j'j}^d \bar{u}_L^i d_R^j - H^- \sum_{i,j' \neq j}^{3} V_{ij}^\dagger \mu_{j'j}^u \bar{d}_L^i u_R^j \Bigg) \\
& + (R+iI) \sum_{i \neq j}^{3} \mu_{ij}^d \bar{d}_L^i d_R^j + (R-iI) \sum_{i \neq j}^{3} \mu_{ij}^u \bar{u}_L^i u_R^j + h.c,
\end{aligned} \tag{10}
$$

where L_1 has no flavor-changing effects other than that expected for $H^\pm$ from the CKM matrix V and L_2 contains the flavor-changing effects for neutral bosons as well as small additional flavor-changing terms for $H^\pm$. The factor $\xi_{f_i} m_{f_i}$ and μ_{ij}^f arise primarily from the diagonal and off-diagonal elements of Γ_i^f respectively.

There are four major sources of CP violation [7,8]: (1) CKM matrix,(2) the phases in factors ξ_{f_i} which provide CP violation in charged-Higgs boson exchange; (3) the phases in μ_{ij}^f, this yields CP violation in FCNC. (4) CP violation in mixing matrix O^H. One of the most distinctive features of these sources is that the factors ξ_{f_i} can provide CP violation in charged Higgs boson exchange in addition to and independent of CKM phase. As a consequence, in $\Delta S = 1$ transitions its contribution to ϵ'/ϵ could be as large as 10^{-3} and become comparable with the experimental data [13,14]. Thus a measurement of ϵ'/ϵ would not necessarily be due to CKM mechanism [7,8,15]. This paper is organized as follows: In section 2, the analysis of the possible constraints of the Yukawa couplings from $F^0 - \bar{F}^0$ mixings is presented. The various sources of CP violation and their influence on the determination of unitarity triangle are discussed in section 3, and section 4 contains our conclusions.

II. CONSTRAINTS FROM $K^0 - \bar{K}^0, B^0 - \bar{B}^0$ AND $D^0 - \bar{D}^0$ MIXINGS

In the standard model, it is known that the neutral meson mixings arise from the box diagram through two-W-boson exchange. The extremely small values of the neutral K and B mass differences impose severe constraints on new physics beyond the SM, especially on those with FCNC at tree level. In the S2HDM, additional contributions to the neutral meson mixings can arise from the box diagrams with charged-scalar exchanges and tree diagrams with neutral-scalar exchanges. The mass difference of $K_L - K_S$ is given by

$$\Delta m_K \simeq 2ReM_{12} \equiv 2Re(M_{12}^{WW} + M_{12}^{HH} + M_{12}^{HW} + M_{12}^{H^0} + M'_{12}) \tag{11}$$

where M_{12}^{WW}, M_{12}^{HH} and M_{12}^{HW} are the contributions from box diagrams through two $W^{\pm}$- boson, two charged-scalar $H^{\pm}$ and one W- boson and one charged-scalar exchanges respectively. $M_{12}^{H^0}$ is the one from the FCNC through neutral scalar exchanges at tree level . M'_{12} presents other possible contributions, such as two-coupled penguin diagrams and nonperturbative effects. They are resulted from the corresponding effective Hamiltonian

$$H_{eff}^{WW} = -\frac{G_F^2}{16\pi^2} m_W^2 \sum_{i,j}^{c,t} \eta_{ij} \lambda_i \lambda_j \sqrt{x_i x_j} B^{WW}(x_i, x_j) \bar{d}\gamma_\mu (1-\gamma_5) s \bar{d}\gamma^\mu (1-\gamma_5) s \tag{12}$$

$$\begin{aligned} H_{eff}^{HH} = & -\frac{G_F^2}{16\pi^2} m_W^2 \sum_{i,j}^{u,c,t} \eta_{ij}^{HH} \lambda_i \lambda_j \frac{1}{4} \{ B_V^{HH}(y_i, y_j) [\sqrt{x_i x_j} \sqrt{y_i y_j} |\xi_i|^2 |\xi_j|^2 \\ & \cdot \bar{d}\gamma_\mu (1-\gamma_5) s \bar{d}\gamma^\mu (1-\gamma_5) s + \sqrt{x_s x_d} \sqrt{y_s y_d} \xi_s^2 \xi_d^{*2} \bar{d}\gamma_\mu (1+\gamma_5) s \bar{d}\gamma^\mu (1+\gamma_5) s \\ & + 2\delta_{ij} \sqrt{x_i x_j} \sqrt{y_s y_d} \xi_s \xi_d^* \xi_i \xi_j^* \bar{d}\gamma_\mu (1+\gamma_5) s \bar{d}\gamma^\mu (1-\gamma_5) s] \\ & + B_S^{HH}(y_i, y_j) \sqrt{x_i y_j} [x_d \xi_d^{*2} \xi_i^* \xi_j^* \bar{d}(1-\gamma_5) s \bar{d}(1-\gamma_5) s \\ & + x_s \xi_s^2 \xi_i \xi_j \bar{d}(1+\gamma_5) s \bar{d}(1+\gamma_5) s + 2\sqrt{x_s x_d} \xi_s \xi_d^* \xi_i \xi_j^* \bar{d}(1+\gamma_5) s \bar{d}(1-\gamma_5) s] \} \end{aligned} \tag{13}$$

$$\begin{aligned} H_{eff}^{HW} = & -\frac{G}{16\pi^2} m_W^2 \sum_{i,j}^{u,c,t} \eta_{ij}^{HW} \lambda_i \lambda_j \{ 2\sqrt{x_i x_j} \sqrt{y_i y_j} \xi_i \xi_j^* B_V^{HW}(y_i, y_j, y_w) \\ & \cdot \bar{d}\gamma_\mu (1-\gamma_5) s \bar{d}\gamma^\mu (1-\gamma_5) s + (y_i + y_j) \sqrt{x_d x_s} \xi_s \xi_d^* [B_T^{HW}(y_i, y_j, y_w) \\ & \cdot \bar{d}\sigma_{\mu\nu} (1-\gamma_5) s \bar{d}\sigma^{\mu\nu} (1+\gamma_5) s + B_S^{HW}(y_i, y_j, y_w) \bar{d}(1-\gamma_5) s \bar{d}(1+\gamma_5) s] \} \end{aligned} \tag{14}$$

where the B^{WW}, B^{HH}_V, B^{HH}_S, B^{HW}_V, B^{HW}_S and B^{HW}_T arise from the loop integrals, and they are the functions of $x_i = m_i^2/m_W^2$ and $y_i = m_i^2/m_H^2$ with $i = u, c, t, W$, their explicit expressions are presented in the Appendix. Here η_{ij}, η^{HH}_{ij} and η^{HW}_{ij} are the possible QCD corrections and $\lambda_i = V_{is}V^*_{id}$. Note that in obtaining above results the external momentum of the d- and s-quark has been neglected. Except this approximation which should be reliable as their current masses are small, we need to keep all the terms. This is because all the couplings λ_i and ξ_i are complex in the model, even if some terms are small, they can still play an important role on CP violation since the observed CP-violating effect in kaon decays is of order 10^{-3}. The contributions of neutral Higgs bosons exchange at tree level can be evaluated by

$$
\begin{aligned}
M^{H^0}_{12} &= < P^0|H^{H^0}_{eff}|\bar{P}^0 > \\
&= \frac{G_F^2}{12\pi^2} f^2_{P^0}\tilde{B}_{P^0} m_{P^0} (\sqrt{\frac{m_{f_i}}{m_{f_j}}})^2 (1+\frac{m_{f_i}}{m_{f_j}})^{-1} m^2_{f'_j} \sum_k (\frac{2\sqrt{3}\pi v m_{P^0}}{m_{H^0_k} m_{f'_j}})^2 (Y^f_{k,ij})^2
\end{aligned}
\tag{15}
$$

with

$$
(Y^f_{k,ij})^2 = (Z^f_{k,ij})^2 + \frac{1}{2} r_{P^0} S^f_{k,ij} S^{f*}_{k,ji}, \qquad Z^f_{k,ij} = -\frac{i}{2}(S^f_{k,ij} - S^{f*}_{k,ji})
$$

$S_{k,ij}$ is related to μ^f_{ij} through:

$$
S^f_{k,ij} = (O^H_{1k} + i\sigma_f O^H_{3k}) \frac{\mu^f_{ij}}{\sqrt{m_i m_j}} \tag{16}
$$

where $\sigma_f = 1$ for d-type quarks, $\sigma_f = -1$ for u-type quarks. The formula is expressed in a form which is convenient in comparison with the one obtained from the box diagram in the SM, here $\sqrt{m_{f_i}/m_{f_j}}$ with convention $i < j$ plays the role of the CKM matrix element V_{ij}, and $m_{f'_j}$ is introduced to correspond to the loop-quark mass of box diagram. Namely f'_j and f_j are the two quarks in the same weak isospin doublet. Note that the result is actually independent of $m_{f'_j}$. Here m_{f_i} are understood as the current quark masses. In our following numerical estimations we will use $m_u = 5.5$ MeV, $m_d = 9$ MeV, $m_s = 180$ MeV, $m_c = 1.4$ GeV and $m_b = 6$ GeV with being defined at a renormalization scale of 1 GeV. f_{P^0} and m_{P^0} are the leptonic decay constant (with normalization $f_\pi = 133$MeV) and the mass of the meson P^0 respectively. $\tilde{B}_{P^0}$ and $\tilde{r}_{P^0}$ are bag parameters defined via

$$
< P^0|(\bar{f}_i(1\pm\gamma_5)f_j)^2|\bar{P}^0 > = -\frac{f_{P^0} m^3_{P^0}}{(m_{f_i}+m_{f_j})^2}\tilde{B}_{P^0} \tag{17}
$$

$$
1 + \tilde{r}_{P^0} = -\frac{< P^0|\bar{f}_i(1\pm\gamma_5)f_j\bar{f}_i(1\mp\gamma_5)f_j|\bar{P}^0 >}{< P^0|\bar{f}_i(1\pm\gamma_5)f_j\bar{f}_i(1\pm\gamma_5)f_j|\bar{P}^0 >} \tag{18}
$$

In the vacuum saturation and factorization approximation with the limit of a large number of colors, we have $\tilde{B}_{P^0} \to 1$ and $\tilde{r}_{P^0} \to 0$, thus $Y^f_{k,ij} = Z^f_{k,ij}$.

It is known that H^{WW}_{eff} contribution to Δm_K is dominated by the c-quark exchange and its value is still uncertain due to the large uncertainties of the hadronic matrix element

$$
< K^0|(\bar{d}\gamma_\mu(1-\gamma_5)s)^2|\bar{K}^0 > = -\frac{8}{3} f^2_K m^2_K B_K \tag{19}
$$

where B_K ranges from 1/3 [16] (by the PCAC and $SU(3)$ symmetry), 3/4 [17] (in the limit of a large number of colors) to 1 [19] (by the vacuum insertion approximation). The results from QCD sum rule and Lattice calculations lie in this range. For small B_K, the short-distance H_{eff}^{WW} contribution to Δm_K fails badly to account for the measured mass difference.

In general, we obtain

$$\Delta m_K = \frac{G_F^2}{6\pi^2} f_K^2 B_K m_K m_c^2 \sin^2\theta \{ \eta_{cc} B^{WW}(x_c) + \frac{1}{4}\eta_{cc}^{HH} y_c |\xi_c|^4 B_V^{HH}(y_c) + 2\eta_{cc}^{HW} y_c |\xi_c|^2 B_V^{HW}(y_c, y_w) + \frac{\tilde{B}_K}{B_K} \sum_k (\frac{2\sqrt{3}\pi v m_K}{m_{H_k^0} m_c})^2 Re(Y_{k,12}^d)^2 \} \tag{20}$$

which is subject to the experimental constraint [18]

$$\Delta m_K = 3.5 \times 10^{-6} eV \sim \sqrt{2}\frac{G_F^2}{6\pi^2} f_K^2 m_K m_c^2 \sin^2\theta \tag{21}$$

The effective Hamiltonian for $B_d^0 - \bar{B}_d^0$ Mixing is calculated with the aid of the box diagrams in full analogy to the treatment of the $K^0 - \bar{K}^0$ system. Its explicit expression can be simply read off from the one for $K^0 - \bar{K}^0$ system by a corresponding replacement: $s \leftrightarrow b$. The "standard approximation" made there, namely neglecting the external momenta of the quarks, is also reliable since dominant contributions come from the intermediate top quark. With this analogy, the considerations and discussions on $K^0 - \bar{K}^0$ mixing can be applied to the $B_d^0 - \bar{B}_d^0$ mixing for the contributions from box diagrams. As it is expected that $|\Gamma_{12}|/2 \ll |M_{12}|$ in the B-system (which is different from K-system), the mass difference for $B_d^0 - \bar{B}_d^0$ system is given by $\Delta m_B \simeq 2|M_{12}|$.

The general form for the mass difference in the $B_d^0 - \bar{B}_d^0$ system can be written

$$\Delta m_B \simeq \frac{G_F^2}{6\pi^2} (f_B \sqrt{B_B \eta_{tt}})^2 m_B m_t^2 |V_{td}|^2 \frac{1}{\eta_{tt}} |\{ \eta_{tt} B^{WW}(x_t) + \frac{1}{4}\eta_{tt}^{HH} y_t |\xi_t|^4 B_V^{HH}(y_t) + 2\eta_{tt}^{HW} y_t |\xi_t|^2 B_V^{HW}(y_t, y_w)] + \frac{\tilde{B}_B}{B_B} \sum_k (\frac{2\sqrt{3}\pi v m_B}{m_{H_k^0} m_t})^2 \frac{m_d}{m_b} \frac{1}{V_{td}^2} (Y_{k,13}^d)^2 \}| \tag{22}$$

which is subject to the experimental constraint [18]

$$\Delta m_B = (3.1 \pm 0.12) \times 10^{-4} eV \sim \frac{G_F^2}{12\pi^2} (125 MeV)^2 m_B (176 GeV)^2 (\sin\theta = 0.22)^6 \tag{23}$$

It is known that in the standard model the short-distance contribution to Δm_D from the box diagram with W-boson exchange is of order of magnitude $\Delta m_D^{Box} \sim O(10^{-9})$ eV, here the external momentum effects have to be considered and were found to suppress the contribution by two orders of magnitude [20]. This is because of the low mass of the intermediate state. It is not difficult to see that the additional box diagram with charged-scalar exchange gives even smaller contribution except $|\xi_s|$ is as large as $|\xi_s| \sim 2m_{H^+}/m_s$ which is unreliable large for the present bound $m_{H^+} > 54.5$ GeV [21]. It has been shown that dominant contribution to Δm_D may come from the long-distance effect since the intermediate states in the box diagram are d- and s-quraks. The original

estimations were found that $\Delta m_D \sim 3\times 10^{-5}$ eV [22] and $\Delta m_D \sim 1\times 10^{-6}$ eV [23]. An alternative calculation [24] using the heavy quark effective theory showed that large cancellations among the intermediate states may occur so that the long-distance standard model contribution to Δm_D is only larger by about one order of magnitude than the short-distance contribution, which was also supported in a subsequent calculation [25].

With this in mind, we now consider the contribution to Δm_D from the neutral scalar interaction in the S2HDM. It is easy to read off from eq.(15)

$$\begin{aligned}\Delta m_D^H = 2|M_{12}^H| &= \frac{G_F^2}{6\pi^2} f_D^2 \tilde{B}_D m_D (\sqrt{\frac{m_u}{m_c}})^2 m_s^2 \sum_k (\frac{2\sqrt{3}\pi v m_D}{m_{H_k^0} m_s})^2 |Y_{k,12}^u|^2 \\ &= 0.64\times 10^{-4} (\frac{f_D\sqrt{\tilde{B}_D}}{210MeV})^2 \sum_{k=1}^{3} (\frac{500GeV}{m_{H_k^0}})^2 \frac{|Y_{k,12}^u|^2}{1}\end{aligned} \quad (24)$$

With the above expected values in the second line for various parameters, the predicted value for Δm_D can be closed to the current experimental limit $|\Delta m_D| < 1.58\times 10^{-4}$ eV [18], this implies that a big $D^0 - \bar{D}^0$ mixing which is larger than the standard model prediction is not excluded. With this analysis, we come to the conclusion that a positive signal of neutral D meson mixing from the future experiments at Fermilab, CESR at Cornell and at a τ-charm factory would be in favor of the S2HDM especially when the exotic neutral scalars are not so heavy.

We now proceed to study the constraints on the parameters of the model. Since the parameters ξ_{f_i} and μ_{ij}^f are in general all free parameters, for simplicity we will consider the constraints in two extreme cases.

Case 1: the mass difference is purely explained through the additional box diagrams from two scalar-boson and one W-boson one scalar-boson. In this case, the parameter ξ_{f_i} is of particular importance. Both its amplitude and phase will play an important role in the neutral meson mass difference and CP violation. It is quite different from the earlier analysis in type 1 and type 2 2HDM [26] in which the three couplings ξ_u, ξ_c, and ξ_t are equal, i.e., $\xi_u = \xi_c = \xi_t = \tan\beta$. This is why the constraint from ϵ is much stronger than the one from Δm_K in those models. In the S2HDM, one has in general $\xi_u \neq \xi_c \neq \xi_t$, there are more degrees of freedom to fit ϵ and Δm_K, Δm_B as well as Δm_D . Since the main contribution to Δm_K comes from c-quark though the loop, the upper bound of ξ_c can be extracted from $K^0 - \bar{K}^0$ mixing. The result is plotted in Fig.1. In the numerical calculations we take $f_K = 161$ MeV and $B_K = 0.75$. The rang of m_H^+ is from 100 to 500 GeV. Since the bound of $|\xi_c|$ strongly depends on the SM prediction on Δm_K, three different values of $\Lambda_{QCD} (\Lambda_{QCD} = 0.21, 0.31, 0.41)$ have been used, the corresponding ratios to the experimental data $(\Delta m_K)_{exp}$ are 0.52,0.67 and 0.91 [27]

In the B system, It is of interest to study its relative ratio to the SM, since large degree of uncertainty can be avoided from CKM matrix $|V_{td}|$ and hardonic matrix elements. In Fig.2 we illustrate the relation between $|\xi_t|$ and the charged Higgs mass m_H^+ when the ratios of the HW and HH box diagram contributions to the WW-box diagram contribution in the SM are 2:1, 1:1 and 0.5:1.

As it is shown in Fig.1 and Fig.2, in general $|\xi_c|$ is much larger than $|\xi_t|$. Even when the HW and HH contributions to Δm_B are twice as much as the SM one, $|\xi_c|$ can still be larger than $|\xi_t|$ by an order of magnitude.

FIGURES

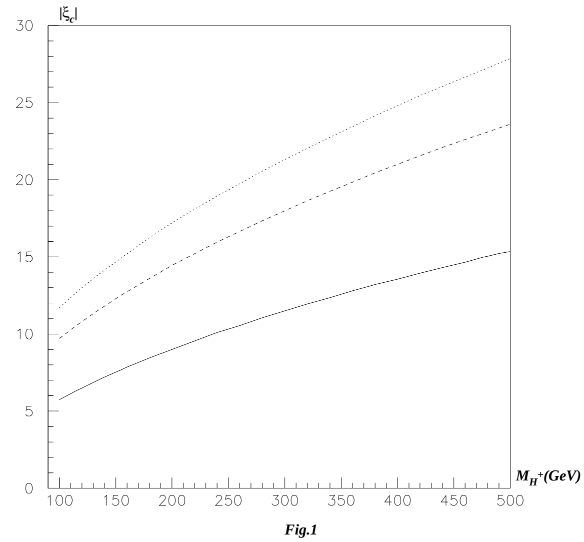

FIG. 1. The upper bound of $|\xi_c|$ with respect to the mass of the charged Higgs scalar in case 1. The three curves are corresponding to the ratios $(\Delta m_K)_{SM}/(\Delta m_K)_{exp} = 0.52$ (dotted), 0.67 (dashed) 0.91 (solid)

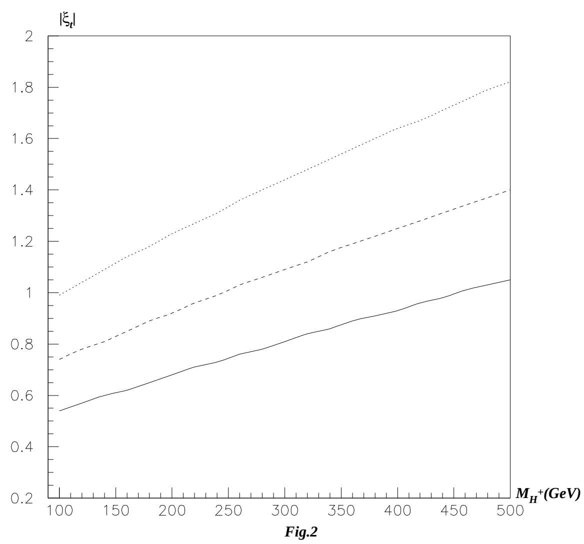

FIG. 2. The value of $|\xi_t|$ with respect to the mass of charged Higgs m_H^+. The three curves are corresponding to the different ratios of HW and HH box diagram to the one from the SM: 2:1 (dotted), 1:1 (dashed), 0.5:1 (solid).

Case 2: the mass difference is fitted through neutral-scalar exchange at tree level. From Eq.(15) we know that the parameters arise in M_{12}^0 are $Y_{k,ij}^f$ rather then μ_{ij}^f. If S_{ij}^f is expected to be symmetric under the exchange $i \leftrightarrow j$ and $r_{P^0} = 1$, $Y_{k,ij}^f$ gets the following simple form

$$Y_{k,i,j}^f = O_{1k}^H \frac{Im\mu_{ij}^f}{\sqrt{m_i m_j}} + \sigma_f O_{3k}^H \frac{Re\mu_{ij}^f}{\sqrt{m_i m_j}} \tag{25}$$

Hence both imaginary and real part of μ_{ij}^f are of importance. Futher more, the phases in μ_{ij}^f also provide a new source of CP violation as we have mentioned in the previous section. To simplify the discussions, we assume that one of the scalar-bosons, for example, the scalar h is much lighter than the other two scalars H and A. The scalar bosons H and A are assumed to be heavier than 500 GeV. The upper bounds can be obtained from $K^0 - \bar{K}^0, B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ mixings. The present consideration is more general than the one in [9] where all the couplings $Y_{k,ij}^f$ are setted to be equal. As a consequence, the constraints from different meson mixings provide different upper bounds upon different $Y_{k,ij}^f$. The results are shown in Fig.3. It is seen from the fig.3 that the upper bound of $Y_{k,12}^u$ is much higher than that from K^0 and B^0 system. This implies that a larger $D^0 - \bar{D}^0$ mixing than the standard model prediction is possible.

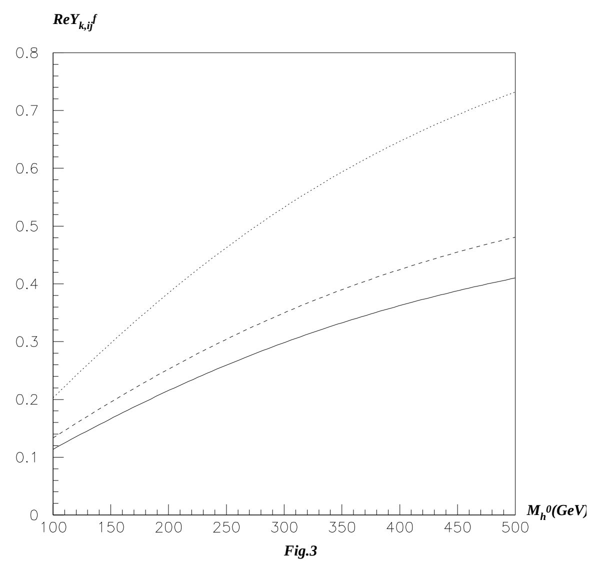

FIG. 3. The m_h^0 dependence of the upper bound of $ReY_{k,ij}^f$. $ReY_{1,12}^d$ from $K^0 - \bar{K}^0$(solid), $ReY_{1,13}^d$ from $B^0 - \bar{B}^0$(dashed) and $ReY_{1,12}^u$ from $D^0 - \bar{D}^0$(dotted). The mass of the other scalar m_A^0 is fixed at $m_A^0 = 500$ GeV.

III. CP VIOLATION AND UNITARITY TRIANGLE

Besides the neutral meson mass difference, the indirect CP violation parameter ϵ_K could also provide constraints on the values of ξ_{f_i} and μ_{ij}^f. The standard definition of ϵ is

$$\epsilon = \frac{1}{\sqrt{2}}(\frac{ImM_{12}}{2ReM_{12}} + \xi_0)e^{i\pi/4} \quad (26)$$

where $\xi_0 = ImA_0/ReA_0$ with $|A_0| = (3.314\pm0.004)\times10^{-7}$ GeV the isospin-zero amplitude of $K \to \pi\pi$ decay. Usually, the ξ_0 term is relatively small as it is proportional to the small direct CP-violating parameter ϵ'.

The first part of contribution to ϵ comes from the box diagram through W-boson and charged-scalar exchanges

$$\begin{aligned} ImM_{12}^{Box} &= ImM_{12}^{WW} + ImM_{12}^{HH} + ImM_{12}^{HW} \\ &= \frac{G^2}{12\pi^2} f_K^2 B_K m_K m_i m_j \{\sum_{i,j}^{c,t} Im(\lambda_i\lambda_j) ReB_{ij}(m_i, m_j; \xi_i, \xi_j) \\ &\quad + Re(\lambda_i\lambda_j) ImB_{ij}(m_i, m_j; \xi_i, \xi_j)\} \end{aligned} \quad (27)$$

where $B_{ij}(m_i, m_j; \xi_i, \xi_j)$ depend on the integral functions of the box diagrams and their general form is given in the Appendix. The imaginary part $ImB_{ij}(m_i, m_j; \xi_i, \xi_j)$ arises from the complex couplings ξ_i.

The second part is due to the flavor changing neutral scalar inteactions at tree level

$$ImM_{12}^{H^0} = \frac{G^2}{12\pi^2} f_K^2 \tilde{B}_K m_K (\sqrt{\frac{m_d}{m_s}})^2 m_c^2 \sum_k (\frac{2\sqrt{3}\pi v m_K}{m_{H_k^0} m_c})^2 Im(Y_{k,12}^d)^2 \quad (28)$$

This provides a contribution to ϵ in almost any models which possess CP-violating flavor changing neutral scalar inteactions.

In particular, the parameter ϵ could receive large contributions from the long-distance dispersive effects through the π, η and η' poles [28]. For a quantitative estimate of these effects, we follow the analyses in refs. [28–31]

$$\begin{aligned} (ImM'_{12})_{LD} &= \frac{1}{4m_K} \sum_i^{\pi,\eta,\eta'} \frac{Im(< K^0|L_{eff}|i >< i|L_{eff}|\bar{K}^0 >)}{m_K^2 - m_\pi^2} \\ &= \frac{1}{4m_K} \frac{2\kappa}{m_K^2 - m_\pi^2} < K^0|L_-|\pi^0 >< \pi^0|L_+|\bar{K}^0 >) \quad (29) \\ &= \frac{G^2}{12\pi^2} f_K^2 B'_K m_K (\frac{m_K}{m_s})^2 \sin\theta m_s^2 (\sqrt{\frac{\pi\alpha_s}{2}} \frac{3\kappa A_{K\pi}}{4m_s(m_K^2 - m_\pi^2)}) \\ &\quad \cdot \sum_i [Im\lambda_i ReP_i^H(m_i, \xi_i) + Re\lambda_i ImP_i^H(m_i, \xi_i)] \quad (30) \end{aligned}$$

where κ is found to be $\kappa \simeq 0.15$ when considering the $SU(3)-$ breaking effects in the $K-\eta_8$ transition, and nonet-symmtry- breaking in $K-\eta_o$ as well as $\eta-\eta'$ mixing. We shall not repeat these analyses, and the reader who is interested in it is refered to the paper [31] and references therein. L_- and L_+ are CP-odd and CP-even lagrangians respectively (with convention $L_{eff} = L_+ + iL_-$). The L_- is induced from the gluon-penguin diagram with charged-scalar

$$L_- = f_s \bar{d}\sigma_{\mu\nu}(1+\gamma_5)\lambda^a s G_{\mu\nu}^a - f_d \bar{d}\sigma_{\mu\nu}(1-\gamma_5)\lambda^a s G_{\mu\nu}^a \quad (31)$$

with

$$f_q = \frac{G}{\sqrt{2}}\frac{g_s}{32\pi^2}m_q \sum_i Im(\xi_q \xi_i \lambda_i) y_i P_T^H(y_i) \tag{32}$$

where $P_T^H(y_i)$ is the integral function and presented in the Appendix. From f_s and f_d it is not difficult to read off the $ReP_i^H(m_i, \xi_i)$ and $ImP_i^H(m_i, \xi_i)$ (see Appendix). In obtaining the last expression of the above equation, we have used the result for the hadronic matrix element $< K^0|L_-|\pi^0 >= (f_s - f_d)A_{K\pi}$ where $A_{K\pi}$ has been computed in the MIT bag model and was found [32] to be $A_{K\pi} = 0.4GeV^3$ for $\alpha_s = 1$, and the convention $< \pi^0|L_+|\bar{K}^0 >= \frac{1}{2}Gf_K^2 B_K' m_K^2 (2m_K/m_s)^2 \sin\theta$, where B_K' is introduced to fit the experimental value $< \pi^0|L_+|\bar{K}^0 >= 2.58 \times 10^{-7}GeV^2$ and is found to be $B_K' = 1.08$. We then obtain $\sqrt{\pi\alpha_s}3\kappa A_{K\pi}/[4\sqrt{2}m_s(m_K^2 - m_\pi^2)] \simeq 1.4$.

Neglecting the $t-$ and $u-$quark contributions and also the terms proportional to m_d in comparison with the terms proportional to m_s, the total contributions to the CP-violating parameter ϵ can be simply calaculated from the following formula

$$\begin{aligned}
|\epsilon| = & 3.2 \times 10^{-3} B_K (\frac{|V_{cb}|}{0.04})^2 \frac{2|V_{ub}|}{|V_{cb}||V_{us}|} \sin\delta_{KM} \{ -\frac{1}{4}[\eta_{cc} B^{WW}(x_c) \\
& + \frac{1}{4}\eta_{cc}^{HH} y_c |\xi_c|^4 B_V^{HH}(y_c) + 2\eta_{cc}^{HW} y_c |\xi_c|^2 B_V^{HW}(y_c, y_w)] \\
& + (\frac{|V_{cb}| m_t}{2m_c})^2 (1 - \frac{|V_{ub}|}{|V_{cb}||V_{us}|}\cos\delta_{KM})[\eta_{tt} B^{WW}(x_t) \\
& + \frac{1}{4}\eta_{tt}^{HH} y_t |\xi_t|^4 B_V^{HH}(y_t) + 2\eta_{tt}^{HW} y_t |\xi_t|^2 B_V^{HW}(y_t, y_w)] \\
& + \frac{m_t}{4m_c}[\eta_{ct} B^{WW}(x_c, x_t) + \frac{1}{2}\eta_{ct}^{HH}\sqrt{y_c y_t}|\xi_c|^2 |\xi_t|^2 B_V^{HH}(y_c, y_t) \\
& + 4\eta_{ct}^{HW}\sqrt{y_c y_t} Re(\xi_c \xi_t) B_V^{HW}(y_c, y_t, y_w)]\} \\
& + 2.27 \times 10^{-3} \frac{Im(\tilde{Y}_{k,12}^d)^2}{6.4 \times 10^{-3}} \tilde{B}_K \sum_k (\frac{10^3 GeV}{m_{H_k^0}})^2 \\
& + 2.27 \times 10^{-3} Im(\xi_c^* \xi_s^*)^2 \frac{6.8GeV^2}{m_{H^+}^2} \tilde{B}_K (ln\frac{m_{H^+}^2}{m_c^2} - 2) \\
& + 2.27 \times 10^{-3} Im(\xi_c \xi_s) \frac{37GeV^2}{m_{H^+}^2} B_K' (ln\frac{m_{H^+}^2}{m_c^2} - \frac{3}{2}) + \frac{\xi_o}{\sqrt{2}}
\end{aligned} \tag{33}$$

where we have used the experimental constraint on $2ReM_{12} = \Delta m_K^{exp.}$

It is analogous to the section 2, we consider the contributions to ϵ in two different cases. The first one, CP violation is governed by the induced KM mechanism, i.e., the first term of the above equation becomes dominant. In this case, new contributions come from the box diagrams of two charged-scalar, and one W-boson and one charged scalar exchanges. Since the expression contains $Re(\xi_c\xi_t)$, the relative phase θ between ξ_c and ξ_t, i.e., $Re(\xi_c\xi_t) = |\xi_c||\xi_t|\cos\theta$, may play an important role. It is of interest to illustrate how such effects can influence the determination of the unitarity triangle. In Fig.4, the constraint to the vertex of the unitarity triangle is given from $|V_{ub}|, \Delta m_B$ and ϵ. Here the new physics effect can change the value of $|V_{td}|$ and the shape of the bounds from ϵ. It is different from the ref. [10] in which $|V_{td}|$ was fixed and taken the value $|V_{td}| = 0.0084$. In our numerical calculations, we take $|\xi_c| = 9.8$ and $|\xi_t| = 0.54$, the mass of charged Higgs is

fixed at $M_H^+ = 200$ GeV. The relative phase between them is taken to be $\pi/3$ and $2\pi/3$ as two examples. The other input parameters are: $B_K = 0.75 \pm 0.15, |V_{ub}|/|V_{cb}| = 0.08 \pm 0.02$ and $|V_{cb}| = 0.04$. For a comparison, we carry out a similar calculation in the SM with the same parameters, the result is plotted in Fig.5. It is found that the shape of the triangle can be largely changed when taking different values of the relative phase between ξ_c and ξ_t. The angle β of the triangle may be extremely small when $\cos\theta$ is close to 1.

The second case, ϵ_K receives contributions from both the charged-scalar and the neutral-scalar exchanges. This case is more important than the one in which only the neutral-scalar exchange is dominant. This is because the relative phase between the the two contributions can largely affect the determination of $|V_{td}|$. To illustrate such phase effect, we choose the ratio between the charged- and neutral-scalar contributions to Δm_B to be 2:1, and take four typical values for the relative phase between them as $0, \pi/3, 2\pi/3$ and π. As it was pointed out by Soares and Wolfenstein [33] that if such phase emerges, then the unitarity angle extracted from $B \to J/\psi K_S$ will be the total phase ϕ_M rather than β. Here ϕ_M is defined by

$$M_{12}^{total} = |M_{12}^{SM} + M_{12}^{NEW}| \exp^{2i\phi_M}$$

with the index 'SM' and 'NEW' denoting the contributions from the SM and new physics. Recently, the preliminary measurement of $B \to J\psi K_S$ was reported by CDF Collaboration [34] with $a_{\psi K_S} = 0.79^{+0.41}_{-0.44}$. However, the resulting constraints on new physics are not very strong [35] due to the large errors. A more precise measurement is expected at B factories.

In Fig.6, we plot the value of V_{td} extracted from Δm_B in the $\rho - \eta$ plane without considering the uncertainty of $B_K (B_K = 0.75)$. The four curves are corresponding to the above four cases. The figure shows that the additional phase from $Y_{k,ij}^f$ can strongly change the value of V_{td}. Its modulus varies in the interval between 0.7 and 1.2 in this situation.

As an example, the influence on the determination of the unitarity triangle in the case 3 (see fig. 6) is plotted in Fig.7. The bounds from ϵ become lower than that from the SM due to the relative phase effect. As the three bounds from $|V_{ub}|, \Delta m_B$ and ϵ still have area in common, the triangle remains to be closed. However, as we have mentioned above, when the angle β is extracted from $B \to J/\psi K_S$, its value will be the total phase ϕ_M which may be much larger. As a consequence, it could make the unitarity triangle to be 'open. This possibility has been shown to happen in the case 3, where $\tan\phi_M$ is three times as large as $\tan\beta$.

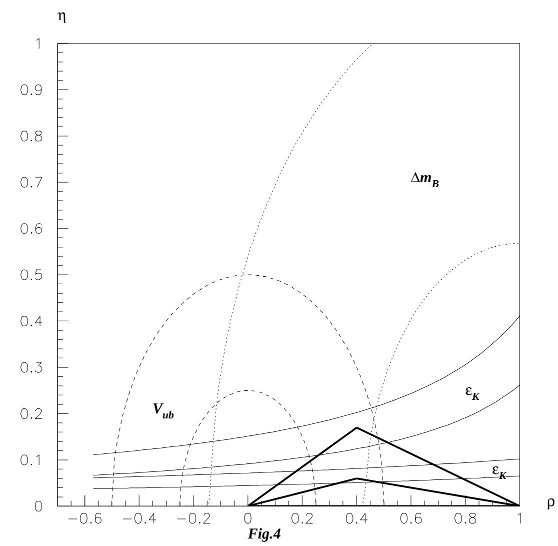

FIG. 4. The constraints on the unitarity triangle in $\eta - \rho$ plane, the two different triangles correspond to $\theta = \pi/3$ (case a) and $\theta = 2\pi/3$ (case b). Where θ is the relative phase between ξ_c and ξ_t. Other parameters are $B_K = 0.75 \pm 0.15, |V_{ub}|/|V_{cb}| = 0.08 \pm 0.02$ and $|V_{cb}| = 0.04$. The allowed region between the two dashed curves are from $|V_{cb}|$, the allowed region between two dotted curves are from Δm_B and the allowed region between two solid curves are from $|\epsilon_K|$ (the two curves below are for the case a for $\theta = \pi/3$ and the two curves above are for the case b for $\theta = 2\pi/3$.)

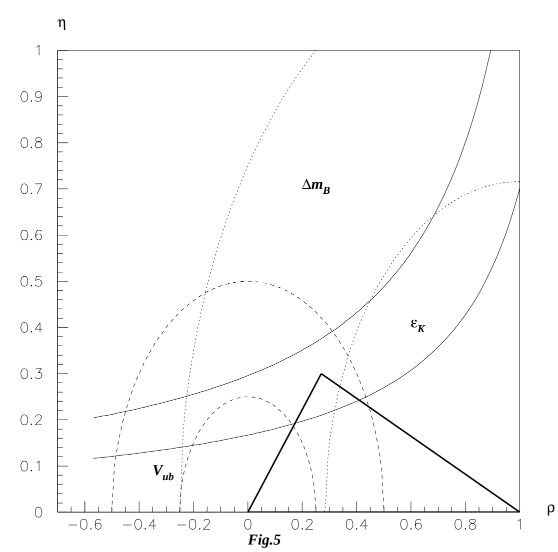

FIG. 5. The constraints on the unitarity triangle from SM. The parameters $B_K, |V_{ub}|, |V_{cb}|$ are as the same as in Fig.4

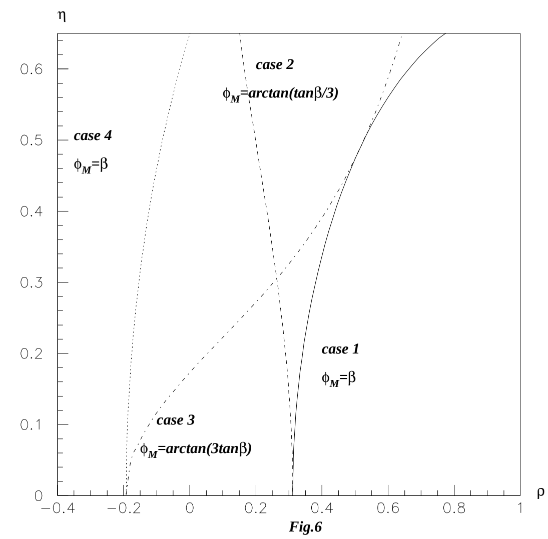

FIG. 6. Constraints on V_{td} from Δm_B. The relative phase between charged- and neutral-scalar exchange is taken to be 0(case 1, solid),$\pi/3$(case 2, dashed),$2\pi/3$(case 3, dash-dotted) and π(case 4, dotted)

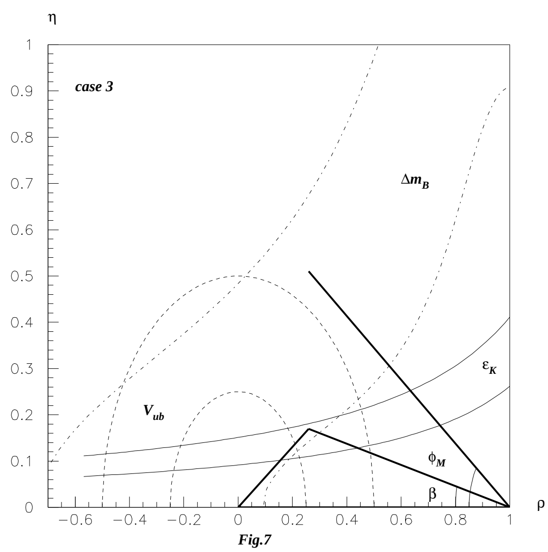

FIG. 7. The constraints on the unitarity triangle in case 3, where $\tan\phi_M$ is three times as large as $\tan\beta$

IV. CONCLUSIONS

In conclusion, we have studied one of the simplest extentions of the standard model with an extra Higgs doublet, which we have simply called as an S2HDM, some constraints on the parameters in the S2HDM have been obtained from $F^0 - \bar{F}^0$ mixing processes. It has been shown that in general $\xi_u \neq \xi_c \neq \xi_t$ and $|\xi_c| \gg |\xi_t|$. A much larger $D^0 - \bar{D}^0$ mixing

than the SM prediction is possible. Various sources of CP violation have been discussed. Their influences on the determination of the unitarity triangle have also been studied in detail. We found that angle β of unitarity triangle could be largely suppressed due to the new contribution from Higgs box diagrams. The phase from neutral Higgs exchange could strongly affect the extraction of β from $B \to J/\psi K_S$. In some cases, such an effect could be so large that the unitarity triangle cannot remain to be closed. In particular, it may even result in the angle β determined from fitting the quantities $|V_{ub}|$, Δm_K and ϵ being different from the one extracted from the decay process $B \to J/\psi K_S$. If it is so, a clear signal of new physics is indicated.

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Appendix

In this appendix, we present some functions and quantities appearing in the text.

A. The integral functions from the box diagrams with W boson and charged scalar exchanges

$$
\begin{aligned}
B^{WW}(x,x') &= \sqrt{xx'}\{\frac{1}{4} + \frac{3}{2}\frac{1}{1-x} - \frac{3}{4}\frac{1}{(1-x)^2}\frac{\ln x}{x-x'} \\
&\quad +(x\leftrightarrow x') - \frac{3}{4}\frac{1}{(1-x)(1-x')}\} \\
B^{WW}(x) &= \frac{1}{4} + \frac{9}{4}\frac{1}{1-x} - \frac{3}{2}\frac{1}{(1-x)^2} + \frac{3}{2}\frac{x^2}{(1-x)^3}\ln x \\
B_V^{HH}(y,y') &= 16\pi^2 m_H^2 I_4 \\
&= \frac{y^2}{(y-y')(1-y)^2}\ln y + (y\leftrightarrow y') + \frac{1}{(1-y)(1-y')} \\
B_V^{HH}(y) &= 16\pi^2 m_H^2 I_1 \\
&= \frac{1+y}{(1-y)^2} + \frac{2y}{(1-y)^3}\ln y \\
B_V^{HW}(y,y',y_W) &= 16\pi^2 m_H^2(\frac{1}{4}I_6 + m_W^2 I_5) = \frac{(y_W - 1/4)\ln y_W}{(1-y)(1-y')(1-y_W)} \\
&\quad -\frac{y(y_W - y/4)}{(y-y')(1-y)(y-y_W)}\ln\frac{y_W}{y} + (y\leftrightarrow y') \\
B_V^{HW}(y,y_W) &= 16\pi^2 m_H^2(\frac{1}{4}I_3 + m_W^2 I_2) = \frac{y_W - 1/4y}{(1-y)(y-y_W)} \\
&\quad +\frac{(y_W - 1/4)}{(1-y)^2(1-y_W)}\ln y + \frac{3}{4}\frac{y_W^2}{(y_W-y)^2(1-y_W)}\ln\frac{y_W}{y} \\
B_S^{HH}(y,y') &= -\frac{y\ln y}{(y-y')(1-y)^2}\ln y - (y\leftrightarrow y') - \frac{1}{(1-y)(1-y')} \\
B_S^{HH}(y) &= -\frac{1}{(1-y)^2}[\frac{1+y}{1-y}\ln y + 2] \\
B_S^{HW}(y,y',y_W) &= 16\pi^2 m_H^2 I_6 = -\frac{\ln y_W}{(1-y)(1-y')(1-y_W)}
\end{aligned}
$$

$$+\frac{y^2}{(y-y')(1-y)(y-y_W)}\ln\frac{y_W}{y}+(y\leftrightarrow y')$$

$$B_S^{HW}(y,y_W)=16\pi^2 m_H^2 I_3=-\frac{y}{(1-y)(y-y_W)}$$

$$-\frac{\ln y}{(1-y)^2(1-y_W)}-\frac{y_W^2}{(y_W-y)^2(1-y_W)}\ln\frac{y_W}{y}$$

$$B_T^{HW}(y,y',y_W)=16\pi^2 m_H^2 m_W^2 I_5=y_W\{\frac{\ln y_W}{(1-y)(1-y')(1-y_W)}$$

$$+\frac{y\ln y/y_W}{(y-y')(1-y)(y-y_W)}+(y\leftrightarrow y')$$

$$B_T^{HW}(y,y_W)=16\pi^2 m_H^2 m_W^2 I_2=y_W\{\frac{1}{(1-y)(y-y_W)}$$

$$+\frac{\ln y}{(1-y)^2(1-y_W)}-\frac{y_W\ln y/y_W}{(y_W-y)^2(1-y_W)}$$

where the functions I_i $(i=1,\cdots,6)$ are the euclidean integrals [36]:

$$I_1(m)=\frac{1}{16\pi^2}\left[\frac{M_H^2+m^2}{M_H^2-m^2)^2}+\frac{2m^2M_H^2}{M_H^2-m^2)^3}\ln\left(\frac{m^2}{M^2}\right)\right]$$

$$I_2(m)=\frac{-1}{16\pi^2M_W^2M_H^2}\left[\frac{M_W^2\ln(M_H^2/m^2)}{(M_H^2-M_W^2)}+\frac{M_W^2\ln(M_W^2/m^2)}{(M_W^2-M_H^2)}+1\right]$$

$$I_3(m)=\frac{1}{16\pi^2}\left[\frac{\ln(M_H^2/M_W^2)}{(M_H^2-M_W^2}\right]$$

$$I_4(m_i,m_j)=\frac{1}{16\pi^2}\left[\frac{m_i^4\ln(m_i^2/M_H)^2}{(m_i^2-m_j^2)(M_H^2-m_i^2)^2}+\frac{m_j^4\ln(m_j^2/M_H)^2}{(m_j^2-m_i^2)(M_H^2-m_j^2)^2}+\frac{M_H^2}{(M_H^2-m_i^2)(M_H^2-m_j^2)}\right]$$

$$I_5(m_i,m_j)=\frac{1}{16\pi^2}\left[\frac{M_H^2\ln(M_W^2/M_H^2)}{(M_H^2-M_W^2)(M_H^2-m_i^2)(M_H^2-m_j^2)}+\frac{m_i^2\ln(M_W^2/m_i^2)}{(m_i^2-M_W^2)(m_i^2-M_H^2)(m_i^2-m_j^2)}\right.$$
$$\left.+\frac{m_j^2\ln(M_W^2/m_j^2)}{(m_j^2-M_W^2)(m_j^2-M_H^2)(m_j^2-m_i^2)}\right]$$

$$I_6(m_i,m_j)=\frac{1}{16\pi^2}\left[\frac{M_H^4\ln(M_H^2/M_W^2)}{(M_H^2-M_W^2)(M_H^2-m_i^2)(M_H^2-m_j^2)}+\frac{m_i^4\ln(m_i^2/M_W^2)}{(m_i^2-M_W^2)(m_i^2-m_H^2)(m_i^2-m_j^2)}\right.$$
$$\left.+\frac{m_j^4\ln(m_j^2/M_W^2)}{(m_j^2-M_W^2)(m_j^2-m_H^2)(m_j^2-m_i^2)}\right]$$

B. The quantities used for calculating the CP-violating parameter ϵ.

$$B_{ij}(m_i,m_j)=\sqrt{y_iy_j}\{\frac{1}{4}|\xi_i|^2|\xi_j|^2B_V^{HH}(y_i,y_j)+2Re(\xi_i\xi_j^*)B_V^{HW}(y_i,y_j,y_W)\}$$
$$+\eta_{ij}B^{WW}(x_i,x_j)+\frac{m_s^2}{4m_im_j}\{\frac{\tilde{B}_K}{B_K}(\frac{m_K}{(m_d+m_s)})^2\sqrt{y_iy_j}B_S^{HH}(y_i,y_j)\xi_{ij}^2$$
$$+2\frac{m_d}{m_s}\sqrt{y_iy_j}B_V^{HH}(y_i,y_j)[\xi_s\xi_d^*\xi_i\xi_j^*+\frac{1}{2}\sqrt{\frac{m_dm_s}{m_im_j}}(\xi_s\xi_d^*)^2]\}$$

with

$$\xi_{ij}^2 = (\xi_i \xi_s - \frac{m_d}{m_s} \xi_i^* \xi_d^*)(\xi_j \xi_s - \frac{m_d}{m_s} \xi_j^* \xi_d^*) - 2\tilde{r}_K \frac{m_d}{m_s} \xi_i \xi_s (\xi_j \xi_d)^*$$

The involved functions arise from the box graphs. From gluonic penguin graph with charged scalar exchange, we have

$$P_T^H(y_i) = \frac{1}{2(1-y_i)} + \frac{1}{(1-y_i)^2} + \frac{1}{(1-y_i)^3} \ln y_i$$

and

$$P_i^H(m_i, \xi_i) = (\xi_s \xi_i - \frac{m_d}{m_s} \xi_d \xi_i) y_i P_T^H(y_i)$$

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